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INFORMS San Antonio 2000 Cluster: Integer Programming

Combinatorial (Bundle) Auctions

Session: SD11
Date/Time: Sunday 15:00-16:30
Type: Invited
Sponsor:
Track:
Cluster: Integer Programming
Room:
Chair: Steef L. van de Velde
Chair Address: Erasmus University, Rotterdam Sch. of Mgmt., PO Box 1738, Rotterdam, 3000 DR , The Netherlands
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SD11.1 Combinatorial Auctions from a Primal-Dual Perspective

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- Rudolf Muller; University of Maastricht, Dept. of Quantitative Econ., Maastricht, 6200 MD , The Netherlands; r.muller@ke.unimaas.nl

An important aspect of the design of combinatorial auctions is the winner determination problem. We take a look at primal-dual algorithms, which share 3 favorable properties: computed solutions are supported by individual item prices, corresponding payment schemes may enforce truth revelation and certificates of optimality of assignments are immediately available.

SD11.2 Complexity & Algorithms for Winner Assignment in Combinatorial Auctions

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We present an analysis of the complexity of the problem to assign bids to bidders in combinatorial auctions. We show that the case of identical assets can be solved in polynomial time. We give some computational results using integer linear programming formulations and heuristics

SD11.3 The Winners Determination Problem in Tendering Transportation Services

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The tendering process for outsourcing transportation of bulk chemicals can be seen as a

A winner determination problem of tendering transportation services

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Abstract

For the tendering of long-term transportation contracts in the bulk industry, shippers use bidbooks that specify for each lane the load location, the destination, the product and the volume that has to be transported over the next so many years. Bidbooks are sent out to a preselected group of carriers, who subsequently quote a price for each of the lanes. After the return of the bidbooks, the shipper determines the winning carriers. Although price is the main driver, a shipper may take into account many additional considerations, including upper bounds on the number of winning carriers in total, per load location, per country of destination, and the maximal transport volume any carrier is allowed to win. The winner determination problem is the problem of finding an allocation of the lanes to the carriers so as to minimize total transportation costs such that each lane is assigned to exactly one carrier and the additional constraints are satisfied. The winner determination problem is extremely hard in practice, and most carriers resort to simple rules of thumb to select the winning carriers. This is no surprise: we prove that the problem is $\mathcal{NP}$-hard in the strong sense. We model the winner determination problem as an integer linear programming (ILP) problem and try and solve the model by use of CPLEX, a state-of-the-art ILP solver. Tested on a comprehensive set of instances generated along the characteristics of a real world case of a European chemical shipper with about 4,000 lanes, CPLEX is capable of solving instances with no more than 270 lanes to optimality, underlining the practical difficulty of the winner determination problem. However, we also develop a fast randomized heuristic, and we show that it performs remarkably well, with a gap of no more than 0.8% from optimality. This performance supports our conclusion that our heuristic can be used to obtain good approximations quickly for realistic, much larger instances.

1 Introduction

For tendering long-term transportation contracts in the bulk industry, shippers use bidbooks that specify for each lane the load location, destination, the product and the volume that has to be transported. The bidbooks are sent out to a preselected group of carriers that meet a certain set of minimal quality requirements. These carriers then quote a price for each of the lanes. After the return of the bidbooks, the shipper determines the winning carriers so as to minimize total transportation costs.

Although price is the main driver in this decision, it is not the only aspect that plays a role. A shipper typically takes into account many additional considerations, as an otherwise inefficient or undesirable situation from an operational and transaction cost point of view may occur. These include an upper bound on the number of winning carriers in total, per load location, per country of destination, and the maximal transport volume any carrier is allowed to win. The problem of allocating lanes to carriers in order to minimize transportation costs subject to such considerations is called the *winner determination problem.*

Price is a major driver in the tendering, because transportation of chemical bulk products is basically a commodity, or in Kraljic (1983)'s terminology, a *leverage product*, which means that the financial impact on the shipping company is high and the risk of not finding a carrier is low. To illustrate this, the last tri-annual tendering of the European chemical company that motivated our research concerned about €100,000,000, whereas the size of its group of preselected carriers was about 30. The role of price makes e-marketplaces and reverse auctions particularly suitable for the sourcing of such transportation services; see for instance Van Weele (2000) for the impact of Internet technology on procurement.

Currently, many transportation e-marketplaces exist, in which shippers and carriers participate to match orders for and bids on transportation services. Ship-

pers participate to find a carrier for certain transportation services and carriers participate to find freight for an otherwise empty return trip. However, most existing transportation marketplaces are used for *spot sourcing*, that is for *short term* transportation contracts, see Kaplan and Sawhney (2000). Examples of independent transportation e-marketplaces are Cargofinder.com, Logistics.com, and Openship.com. Most of these marketplaces do not offer services to facilitate shippers' *systematic sourcing*, that is long term transportation contracts. A notable exception is Translogistica.com. In our paper, we focus on the systematic sourcing of transportation services.

The procurement of transportation contracts is extensively described by Caplice (1996), who also explores the possibilities for operating a reverse auction for tendering transportation services. Furthermore, he presents various carrier assignment models that include some of the considerations we pointed out earlier. Ledyard et al. (2002) describe a so-called combined-value auction, that is, a combinatorial auction for transportation services for Sears, Roebuck and Co., but hardly no attention is paid to modeling and algorithmic aspects. In business practice, Logistics.com and CombineNet.com have shown that an auction of transportation services of this type is really beneficial.

Our work is motivated by a European chemical company that explored the possibilities of having an electronic reverse auction for tendering transportation services. Just a virtual bidbook on the Internet where carriers can submit their bids, is not sufficient to operate such a reverse auction. Other issues that have to be addressed before operating such an auction include the design of the auction mechanism (for instance, single round versus multiple-round; see Cramton (1998) and the references therein) and the solution of the winner determination problem.

Approximating rather than optimizing the winner determination problem has consequences for the economic efficiency of the auction. Nisan and Ronen (2000) show that under a VCG-mechanism a combinatorial auction is truthful if and only

if the winner determination problem is solved to optimality. A truthful mechanism maximizes the total welfare, and hence approximating the winner determination problem can lead to a lower social welfare and revenue loss.

Accordingly, there is a growing body of literature on optimization algorithms for the winner determination problem, in particular for so-called combinatorial auctions, where bidders are allowed to bid XOR bids (I want X or Y but not both). Sandholm (2002) developed a search algorithm to solve the winner determination in a combinatorial auction to optimality. The search generates each allocation with positive revenue exactly once, which results in a worst case complexity of $O(n^m)$, where n is the number of bids and m the number of items on sale. Fujishima, Leyton-Brown, and Shoham (1999) present a depth-first search algorithm with a number of speed-ups for the winner determination problem in combinatorial auctions. Davenport and Kalagnanam (2002) present a volume discount and a combinatorial auction mechanism for the procurement of direct inputs of a manufacturing firm. For both auctions they present a mathematical programming model of the winner determination problem. The models take into account several of the side constraints that are also present in our problem, such as the maximum number of suppliers and the maximum quantity to be allocated to any supplier. Andersson et al. (2000) present a mixed integer linear programming formulation for this winner determination problem, which can be solved by standard mixed integer linear programming solvers. They compare this approach with existing algorithms on different problem distributions and conclude that such a standard approach performs quite reasonably. Bikhchandani and Ostroy (2001) present a number of different extended linear programming formulations for the winner determination problem, one of which always has an integral solution. Bikhchandani et al. (2002) propose another extended integral formulation with fewer variables. Nisan (2000) suggests a linear programming-based approach for solving the winner determination problem under different bidding languages. The method first solves a linear program, af-

ter which branch-and-bound is applied to find a feasible assignment. Günlük et al. (2002) present an alternative integer linear programming formulation based on a set packing formulation with a larger number of variables, which gives rise to a tighter linear programming relaxation. In this formulation, variables indicate whether a bundle is assigned to a bidder or not. The authors present also a branch-and-price algorithm, which served as shadow-solver in one of the FCC-auctions. Hoos and Boutilier (2000) present a stochastic local search algorithm for the winner determination problem and show that this method finds high quality solution much faster than current optimal methods. Zurel and Nisan (2001) present an approximation algorithm for the winner determination problem in a combinatorial auction that consists of approximating the linear programming relaxation and then applying a sequence of greedy hill-climbing algorithms to improve on the initial solution. The authors claim an average approximation error of less than 1% for a variety of problem instances. For an extensive overview of the winner determination problem in combinatorial auctions, we refer to De Vries and Vohra (2003).

In our problem, the bids are OR-bids for single lanes, and not XOR. The complicating feature of our problem lies in the many combinatorial constraints on the possible allocations to the carriers. These two aspects differentiate our problem from the existing literature. Furthermore, we have access to the real data of a very large tendering of transportation services.

Indeed, the winner determination problem is extremely hard in practice, and most carriers resort to simple rules of thumb to select the winning carriers. This is not surprising: the problem is $\mathcal{NP}$-hard in the strong sense. We model the winner determination problem with its many practical side constraints as an integer linear programming (ILP) problem. Tested on a comprehensive set of instances generated along the characteristics of a real world case of a European chemical shipper with about 4,000 lanes, CPLEX, a state-of-the-art ILP solver, is capable of solving instances with no more than 270 lanes to optimality, underlining the

practical difficulty of the winner determination problem. However, we also develop a fast randomized heuristic, and we show that it performs remarkably well.

The theoretical and practical relevance of our findings is multifold. First, we prove that the winner determination problem with OR-bids of single items together with many side-constraints is $\mathcal{NP}$-hard in the strong sense and show also that general optimization technology (such as CPLEX) is unable to solve the very large instances of the winner determination problem to optimality; accordingly, if we really wish to apply such technology, then we need to reduce the instance size by bundling certain lanes (like Ledyard et al. (2002) have done). Second, the linear programming relaxation of our model formulation gives very strong lower bounds, on average no more than 0.6% away from optimal. Third, a reasonably simple randomized heuristic performs remarkably well, within about 0.8% from optimality. Hence, an alternative approach to tackle large, realistic instances would be to use the presented heuristic, to run it more often, or to develop a more sophisticated version. Finally, we have shown that substantial savings are possible using algorithmic support for the winner determination problem; this confirms similar findings by Ledyard et al. (2002) for the Sears, Roebuck and Co. case.

The outline of the paper is as follows. In Section 2 we describe the problem. In Section 3 we formulate the winner determination problem as an integer linear programming problem. Section 4 describes our randomized heuristic. Computational results are discussed in Section 5. Section 6 gives the conclusions of the paper.

2 Problem description

We consider a transportation tendering process, in which each lane is described by the load location, the destination, and the product and volume that have to be transported over a pre-specified period of time. Assume T transportation companies, referred to as carriers, are bidding on the L lanes. The lanes have destina-

tions in C different countries and have to be transported from O origins, called load locations. The total volume of the products to be transported is v and let v_i be the volume that has to be transported for lane i $(i = 1, \ldots, L)$. Let b_{ij} $(i = 1, \ldots, L, j = 1, \ldots, T)$ be the bid of carrier j on lane i. We assume that all bids are positive integers. If carrier j did not bid on lane i, then $b_{ij} = \infty$. The shipper wants no more than MC winning carriers in total. Analogously, no more than c_k carriers are allowed to win lanes to country k $(k = 1, \ldots, C)$, and no more than p_h carriers from load location h, $(h = 1, \ldots, O)$. In order not to become too dependent on a single carrier, no carrier may receive a volume of more than the fraction q of the total transport volume v.

We assume that each carrier has sufficient transportation capacity for the lanes it wins. If needed, capacity constraints for each carrier could easily be included in the model. A feasible allocation is one such that every lane is allocated to exactly one carrier, each carrier receives a set of lanes with a total volume no more than qv, the number of winning carriers is at most MC, for each country k $(k = 1, \ldots, C)$ there are no more than c_k winning carriers, and for each load location h $(h = 1, \ldots, O)$ there are no more than p_h winning carriers. Depending on the model parameters it is possible that no feasible allocation exists. If no feasible allocation exists, the shipper typically would increase MC, c_k or p_h for some k or h to enlarge the feasible region. In a typical tendering process, the shipper plays with these parameters to evaluate different scenarios. The objective is to find a feasible allocation with minimal transportation costs.

3 Mathematical formulation

We formulate the winner determination problem as an integer linear programming problem. Let z_{ij} be a binary variable that adopts the value 1 if carrier j is allocated lane i and the value 0 otherwise. Let $y_j = 1$ if carrier j is allocated at least one

lane; $y_j = 0$, otherwise. $c_j^k = 1$ if carrier j gets at least one lane to country k and $c_j^k = 0$, otherwise. Let $p_j^n = 1$ if carrier j gets at least one lane from load location n and $p_j^n = 0$, otherwise.

The goal is to find an allocation of lanes to carriers such that every lane is allocated to exactly one carrier, all constraints are satisfied and transportation costs are minimized. Mathematically, the winner determination problem, in the remainder referred to as problem WD, is to find values for the decision variables y_j, z_{ij}, p_j^h, and c_j^k that minimize

$$
\begin{array}{llll}
\text{minimize} & \displaystyle\sum_{j=1}^{T}\sum_{i=1}^{L} b_{ij} z_{ij} & & \\
\text{subject to} & z_{ij} - y_j \leq 0 & i = 1,\ldots,L, j = 1,\ldots,T & (1) \\
& \displaystyle\sum_{j=1}^{T} y_j \leq MC & & (2) \\
& \displaystyle\sum_{j=1}^{T} z_{ij} \geq 1 & i = 1,\ldots,L, & (3) \\
& \displaystyle\sum_{i=1}^{L} v_i z_{ij} \leq qv & j = 1,\ldots,T, & (4) \\
& a_{ik} z_{ij} \leq c_j^k & i = 1,\ldots,L, j = 1,\ldots,T, k = 1,\ldots,C & (5) \\
& \displaystyle\sum_{j=1}^{T} c_j^k \leq c_k & k = 1,\ldots,C, & (6) \\
& h_{in} z_{ij} \leq p_j^h & i = 1,\ldots,L, j = 1,\ldots,T, h = 1,\ldots,O & (7) \\
& \displaystyle\sum_{j=1}^{T} p_j^h \leq p_h & h = 1,\ldots,O, & (8) \\
& z_{ij}, y_j, c_j^k, p_j^h \in \{0,1\} & i = 1,\ldots,L. j = 1,\ldots,T. & (9)
\end{array}
$$

Constraints (1) ensure that lanes are only allocated to one of the MC winning carriers. Constraint (2) ensures that there are no more than MC winning carriers. Constraints (3) guarantee that each lane is allocated at least once. Constraints

(4) enforce that each carrier is allowed to transport no more than qv in total. Constraints (5) ensure that lanes to country k cannot be allocated to carriers that are not allowed to transport to country k. Constraints (7) are similar constraints for the load locations. Constraints (6) and (8) enforce that there are no more than c_k winning carriers for each country k ($k = 1, \ldots, C$), and no more than p_h carriers for each load location h ($h = 1, \ldots, O$). Constraints (9) are the integrality constraints, ensuring that lanes are not split over different carriers.

The linear programming relaxation is obtained by relaxing constraints (9). The solution of the linear programming relaxation is a lower bound on the optimal solution value of WD.

Next, we prove that the winner determination problem is $\mathcal{NP}$-hard in the strong sense.

Theorem 1. *The winner determination problem WD is $\mathcal{NP}$-hard in the strong sense.*

Proof. The decision variant DWD of WD is the problem: Does there exist an allocation of the lanes to the carriers satisfying constraints (1) to (9)? If DWD is $\mathcal{NP}$-complete in the strong sense, WD is $\mathcal{NP}$-hard in the strong sense. To prove that DWD is $\mathcal{NP}$-complete in the strong sense, we present a polynomial reduction from 3-PARTITION, which is known to be $\mathcal{NP}$-complete in the strong sense (Garey and Johnson (1979)), to DWD. The problem 3-PARTITION is defined as follows

Instance: A set $Q = (q_1, \ldots, q_{3m})$ of $3m$ elements, a bound $B \in \mathbb{Z}^+$ and a size $s_q \in \mathbb{Z}^+$ for each $q \in Q$ such that $\frac{B}{4} < s_q < \frac{B}{2}$ and such that $\sum_{q \in Q} s_q = mB$

Question: Can Q be partitioned into m disjoint sets $Q_1, \ldots, Q_m$ such that $\sum_{q \in Q_i} s_q = B$ for each $i = 1, \ldots, m$?

It is clear that $DWD \in \mathcal{NP}$. We will prove that an instance of DWD is a yes-

instance if and only if the corresponding instance of 3-PARTITION is a yes-instance.

For any given instance of 3-PARTITION we construct the following instance of DWD

$$
\begin{aligned}
T &= m, \\
L &= 3T, \\
v &= mB, \\
q &= \frac{1}{m}, \\
v_i &= s_i, \text{ for } i = 1, \ldots, L, \\
b_{ij} &= 1, \text{ for } i = 1, \ldots, L, j = 1, \ldots, T, \\
c_k &= T, \text{ for } k = 1, \ldots, C, \\
p_h &= T, \text{ for } h = 1, \ldots, O, \\
MC &= T, \\
k &= 3m.
\end{aligned}
$$

Note that $\frac{qv}{4} < v_i < \frac{qv}{2}$. It is obvious that the transformation is polynomial. In the winner determination problem instance we have m carriers, $3m$ lanes, each carrier can be allocated a volume of maximal B. Each carrier is allowed to transport to each country and from each load location.

Consider an arbitrary yes-instance of 3-PARTITION. Hence, the set Q of elements can be partitioned into m disjoint subsets $Q_1, \ldots, Q_m$ such that for $i = 1, \ldots, m$ holds $\sum_{q \in Q_i} s_q = B$. For the corresponding instance of DWD this means that the $3T$ lanes can be divided into T disjoint sets $A_1, \ldots, A_T$ such that for each $i = 1, \ldots, T$ holds $\sum_{j \in A_i} v_j = qv$. Hence, we can assign to every carrier a bundle of three lanes with volume qv. This assignment clearly satisfies all constraints. This proves that each yes-instance of 3-PARTITION corresponds to a yes-instance of DWD.

Next we prove the other implication of the equivalence. Consider a yes-instance of DWD that is obtained from a 3-PARTITION instance. In that case, there is an assignment satisfying all constraints. Each carrier gets a volume of maximal qv. Because $mqv = v$, he gets exactly qv. Because $v_i > \frac{qv}{4}$, there cannot be four lanes or more in a bundle. Since there are m carriers and $3m$ lanes, every carrier gets exactly three lanes. This assignment can be transformed to a solution of the corresponding 3-PARTITION problem. □

4 Randomized heuristic

In this section we develop a simple randomized heuristic to find a good feasible solution. Note that without loss of generality, we may assume that every carrier bids on each lane. After all, if a certain carrier j did not bid on lane i, then we simply let $b_{ij} = \infty$. This assumption makes it easier to find feasible solutions.

We apply the heuristic a pre-specified number of times. The heuristic is as follows

Step 1 Randomly select a carrier. Assign to this carrier each lane for which its bid is lowest among all carriers, if the volume of the bundle after adding the lane is smaller than or equal to qv and constraints (6) and (8) are not violated by the assignment.

Step 2 If all lanes have been assigned: STOP.

Step 3 If less than MC carriers have been selected: go to step 1.

Step 4 For each unassigned lane, determine the set of selected carriers to which this lane can be allocated without violating any of the constraints. If this set is empty, then abort the heuristic - a feasible solution cannot be found. Otherwise, allocate the lane to the carrier with the lowest bid among the ones that can be allocated the lane without violating the constraints.

In theory, it is $\mathcal{NP}$-hard to find a feasible allocation as we have shown in Theorem 1. However, in practice the heuristic is very fast and if q, c_k $(k = 1, \dots, C)$ and p_h $(h = 1, \dots, O)$ are reasonable large, it is likely that a feasible allocation will be found. Note that the heuristic is likely to give a different allocation each time it is run, because of its randomized nature.

5 Computational results

The algorithm was coded in C++. The integer linear programs were solved by the software package CPLEX, (ILOG, 2001), version 7.1. The tests were performed on a 1000 Mhz PC with 200 MB RAM.

We tested the empirical performance of the integer linear programming approach and the heuristic on a comprehensive set of instances drawn from a real-life case for a large European chemical shipper. In this data set, there are 10 carriers, destinations in 27 countries, 10 load locations, 4,000 lanes with the volume per lane between 10 and 5,000 tons, and the price per ton for each lane between €20 and €500 depending on the distance between the load location and the destination and the type of product that has to be transported. Since the market for transportation of chemical bulk products is very competitive, bids for a lane do not differ much.

We used the following method to generate random instances of the winner determination problem

Step 1 For each lane i the volume v_i is randomly generated uniformly from the integers between 10 and 5,000.

Step 2 For each lane i the average price per ton r_i is randomly generated uniformly from the integers between 20 and 500.

Step 3 The price per ton for lane i by carrier j is randomly generated uniformly from the integers between $\lfloor \frac{9r_i}{10} \rfloor$ and $\lfloor \frac{11r_i}{10} \rfloor$. The total bid of carrier j for

lane i equals this price multiplied by v_i.

For all instances, the maximal fraction q of the transport volume is 0.6 and the maximal number of carriers for each load location and country is 2 ($c_k = 2$, $k = 1, \ldots, C$ and $p_h = 2$, $h = 1, \ldots, O$). Instances are generated for $MC = 3, 4$ and 5 and $T = 5$ and $T = 10$. Instances with 25 countries and 2 load locations, with 20 countries and 3 load locations and with 27 countries and 10 load locations are considered. For each set of parameters we generated 25 instances.

We report the average and maximal relative gap between the best found heuristic solution and the optimal linear programming solution, the average and maximal relative gap between the best found heuristic solution and the optimal integer linear programming solution, the number of instances out of 25 for which the optimal linear programming relaxation solution was integer, the average and maximal computation time, the average and maximal time needed to solve the linear programming relaxation and the average and maximal number of nodes in the branch-and-bound tree; see Table 1 for a summary of our notation. The computational results are given in Table 2. For each set of instances the optimal solution of the linear programming relaxation is found within 30 seconds on average.

The instances with ten carriers take on average about twice as much computation time as the instances with five carriers. In part, the difference in computation times can be explained by 28% of the instances with five carriers having integral solutions, against only 9% of the instances with ten carriers. We found no direct relationship between MC, the number of carriers allowed, and the difficulty of an instance. For the instances with $C = 27$, $L = 10$ and $T = 5$, the heuristic gives a solution equal to the optimal integer linear programming solution for 18 instances if $MC = 3$, for 19 out of the 25 instances if $MC = 4$, and for 18 out of the 25 instances if $MC = 5$. So, our heuristic succeeds in finding an optimal solution quite often, which fuels our belief that our heuristic might even work better for larger instances.

We compute the relative gap as the difference between the heuristic solution value and the linear programming lower bound divided by the linear programming lower bound. For our heuristic, this gap was only 0.8% on average with an average computation time of less than 2 seconds.

We also ran CPLEX with the solution found by our heuristic as an initial solution for the instances with $C = 27$, $O = 10$ and $T = 5$. These results are given in Table 3. The average total running times are somewhat smaller; for the smaller instances, the improvement is even less.

6 Conclusion and future research

Sodhi (2001) stresses the importance of Operations Research for improving the efficiency of supply chains, as it can be used in functionalities for facilitating procurement and planning decisions which can be offered by business-to-business e-marketplaces additional to the spot sourcing services. Keskinocak and Tayur (2001) and Geoffrion and Krishnan (2001) point out that algorithmic support is needed to match orders and bids in e-marketplaces.

In this paper, we have tried to contribute to this area by addressing a winner determination problem in a reverse auction (without XOR bids) for the tendering of transportation services with many practical and complicating side-constraints. The disappointing performance of the optimization method stresses that still many challenges lie ahead for the Operations Research community. Whereas the chemical shipper has about 4,000 lanes with 10 load locations to auction off, CPLEX can solve instances with no more than only 270 lanes with ten load locations to optimality.

However, our simple randomized heuristic gives surprisingly good results, with the average solution quality no more than 0.8% away from optimal. Better performance may be expected from a more sophisticated heuristic, for example randomized local search. Hence, for practical purposes, the design of such randomized

heuristics seems to be a very promising avenue for further research.

In our case, the shipper disallowed XOR bids. However, in practice, the carriers value a bundle of lanes not always as the sum of the values of the individual lanes. A possible research direction is the analysis of a model with XOR-bids. The winner determination problem with XOR bids has been extensively analyzed (Sandholm (2002) and Günlük et al. (2002)), without the much complicating constraints specific for the transportation sector, however.

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LP^*	objective value of the optimal linear programming solution
ALH	average gap between the best heuristic solution and LP^*
MLH	maximal gap between the best heuristic solution and LP^*
AHI	average gap between the best heuristic solution and the optimal integer solution
MHI	maximal gap between the best heuristic solution and the optimal integer solution
I	instances out of 25 with integer optimal LP solution
#HO	instances out of 25 with heuristic solution equal to optimal solution
AT	average total computation time in seconds
MT	maximal total computation time in seconds
AL	average computation time for solving LP relaxation
ML	maximal computation time for solving LP relaxation
AN	average number of nodes in the branch-and-bound tree
MN	maximal number of nodes in the branch-and-bound tree

Table 1: Notation used in reporting the computational results

MC	C	O	T	ALH	MLH	#HO	AHI	MHI	I	AT	MT	AL	ML	AN	MN
3	25	2	5	0.4	0.9	3	0.4	0.8	9	0.59	2.31	0.11	0.14	1.24	8
			10	0.5	1.0	3	0.4	0.8	2	7.79	20.15	0.88	1.29	3.12	16
	20	3	5	0.5	1.2	1	0.4	1.2	10	2.71	7.16	0.24	0.35	1.84	9
			10	0.6	1.0	0	0.4	0.9	4	48.46	97.27	1.59	2.67	26.64	126
	27	10	5	0.4	0.6	18	0.01	0.1	0	1093	2380	16.61	26.37	1120	3220
	27		10	0.8	1.1	-	-	-	0	-	-	90.24	172.1	-	-
4	25	2	5	0.4	1.0	2	0.3	1.0	7	0.45	1.77	0.11	0.16	0.92	4
			10	0.6	1.0	1	0.5	1.0	5	4.74	16.29	0.79	1.00	3.16	12
	20	3	5	0.5	1.2	1	0.4	1.2	8	2.60	9.20	0.25	0.35	4.12	36
			10	0.8	1.1	0	0.6	1.1	2	41.48	79.28	1.40	2.16	27.36	154
	27	10	5	0.4	1.0	19	0.03	0.4	0	1184	3542	18.02	33.32	1413	7151
	27		10	0.9	1.3	-	-	-	0	-	-	81.79	130.7	-	-
5	25	2	5	0.5	1.1	1	0.4	1.1	14	0.31	1.56	0.07	0.1	1.32	10
			10	0.6	1.1	1	0.5	1.0	6	3.14	9.88	0.71	1.12	2.44	13
	20	3	5	0.4	1.1	1	0.4	1.1	16	0.98	6.22	0.14	0.18	1.64	12
			10	0.7	1.1	0	0.5	1.0	1	41.80	76.95	1.46	2.11	24.84	142
	27	10	5	0.4	0.6	18	0.03	0.2	0	1394	6182	15.76	25.57	1894	11949
	27		10	0.9	1.2	-	-	-	0	-	-	72.31	104.8	-	-
6	27	10	10	0.9	1.2	-	-	-	0	-	-	72.64	112.8	-	-

Table 2: Computational results for the case MC=3, MC=4 and MC=5

MC	C	O	T	ALH	MLH	I	AT	MT	AL	ML	AN	MN
3	27	10	5	0.4	0.6	0	1076	2647	17.16	26.96	1278	5151
4	27	10	5	0.4	1.0	0	1041	3262	16.08	31.27	1446	6242
5	27	10	5	0.4	0.6	0	1276	5337	15.76	26.59	1818	10670

Table 3: Computational results for the case MC=3, MC=4 and MC=5 using initial solutions